Submitted October 2002

Guided Wave Simulation and Visualization by a Semianalytical Finite Element Method

by Takahiro Hayashi* and Joseph L. Rose†

ABSTRACT

Simulation and visualization of guided wave propagation can be very useful in both educational and research studies. However, even with the tremendous computational power available today, calculation time and memory have become major issues due to large calculations corresponding to the guided wave testing area. To overcome these problems, we employ a semianalytical finite element method for simulation of guided waves in a plate and a pipe. Two major characteristics of a guided wave, dispersion and multimode existence, are demonstrated in visualization results. Moreover, guided wave propagation in a pipe is discussed for three cases: axisymmetric guided modes in a straight pipe; focusing effect due to tuning time delays and amplitudes; and axisymmetric input for a pipe with an elbow.

Keywords: guided wave, simulation, visualization, semianalytical finite element method.

INTRODUCTION

Background

Since guided waves propagate over long distances by tuning modes and frequency appropriately, guided wave testing has excellent overall discontinuity detection potential. However, the peculiar characteristics of multimode existence and dispersion can make wave mechanics complex and analyses difficult. Simulation and visualization of guided wave propagation can therefore play an important role in both educational and research studies of discontinuity detection.

When lamb wave excitation methods in a plate are considered, for example, transducer geometries such as the incident angle and diameter of an angle beam transducer or the element number and spacing width of a comb type transducer must be determined. Design parameters are based on phase velocity and wavelength determined from the material's structural geometry and properties. Topics in utilizing dispersion curves have been presented in many textbooks (Viktrov, 1967; Achenbach, 1984; Kino, 1987; Auld, 1990; Graff, 1991; Rose, 1999) and have become commonplace during this past decade. Despite the detailed and elaborate explanations with equations and illustrations, guided wave mechanics are still complex. Guided wave simulation and visualization is helpful in the understanding and appreciation of such complicated wave mechanics.

Guided wave interaction with discontinuities or elbows, which are not yet fully theoretically clarified, has become crucial in plate and pipe testing. Wave motion of flexural modes excited by a partial loading on a pipe, for example, are complex but quite useful. Guided wave simulations and visualizations reveal such guided wave mechanics principles and provide new data acquisition and analysis approaches to nondestructive testing.

Method

Visualization of ultrasonic wave propagation in a solid was experimentally carried out by several investigators. Photographs of ultrasonic wave propagation in a glass plate using dynamic photoelasticity are shown in Zhang et al. (1988) and Li and Negishi (1994). For pioneering efforts on numerical simulation and visualization, see Harumi (1986) and Yamawaki and Saito (1992), who calculated and visualized bulk wave propagation. Now numerical simulation for guided waves is possible.

To carry out guided wave simulation and visualization, waveform numerical data are required at many grid points covering the visualization region. Suppose that an $A0$ mode of 3 mm/µs (0.1 in./µs) at a frequency of 1 MHz is visualized. While the grid point width needs to be sufficiently smaller than the wavelength of 3 mm (0.1 in.), a visualization region (in other words, a testing area in practice) is usually in the several centimeter or meter order. At least several hundred or thousand grid points are required in the propagation direction. Two grid points are necessary as a minimum in the thickness direction and more grid points are necessary for higher order modes with complicated wave structure. Moreover, waveform data at each point should consist of a data series with sufficiently small time steps corresponding to sampling frequency several times as high as center frequency. Consequently, several hundred or thousand data are required in the time direction, too.

Recently, such a large calculation has become feasible with sufficient accuracy to obtain very useful results for NDT by the use of commercial general purpose software (Demma et al., 2001; Sanderson and Smith, 2002). On the other hand, specialized techniques for guided wave calculation such as a hybrid method and a semianalytical finite element method have been developed to avoid large calculation time and memory. In the hybrid technique, semiinfinite regions are covered by the normal expansion theory and an arbitrary shape region between two semiinfinite regions, such as a crack region, is calculated by the finite element method or boundary element method. Since the number of nodes in finite element method or boundary element method regions is related to calculation time, hybrid methods with a small finite element method or boundary element method region require much less calculation time. This technique is efficient for calculations of guided waves in a plate and in a straight pipe where wave motions in semiinfinite

* Department of Mechanical Engineering, Nagoya Institute of Technology, Gokiso Showa Nagoya 466-8555, Japan; 81 52 735 5338; fax 81 52 735 5342; e-mail <hayashi@megw.mech.nitech.ac.jp>.
† Department of Engineering Science and Mechanics, Pennsylvania State University, 212 Earth and Engineering Building, University Park, PA 16802; (814) 863-8026; fax (814) 863-8164; e-mail <jlresm@engr.psu.edu>.

regions can be expressed by the normal expansion theory. (See Cho and Rose [1996] for the hybrid boundary element method, Al-Nassar [1991] for the hybrid finite element method and Ditri and Rose [1992] for a pipe issue.) In the semianalytical finite element method, subdivisions in the propagating direction of the guided waves are not needed due to the use of orthogonal functions in the propagating direction. Reducing one finite element method dimension enables us to calculate guided wave propagation characteristics with faster speed and less memory. This technique is feasible for guided waves in a pipe and in a bar with an arbitrary cross section, such as a rail, as well as for lamb waves in a plate. (See detailed studies for lamb wave calculation in Liu and Achenbach [1995], Hayashi et al. [2002], Galán and Abascal [2002], Zhuang et al. [1999], Hayashi et al. [in press (a)] for pipe and Taweel et al. [2000] and Hayashi et al. [in press (b)] for a bar with an arbitrary cross section.)

In this paper, simulation and visualization of guided wave propagation in a plate and a pipe are carried out using the semianalytical finite element method. Dispersion and multimode existence of the fundamental lamb modes are discussed. Moreover, wave mechanics in a straight pipe and a pipe with an elbow are visualized.

LAMB WAVE SIMULATION

In the semianalytical finite element method, the cross section is divided only in the thickness Y direction. Waves propagating in the longitudinal X direction are expressed by an orthogonal function exp $(i\xi x)$, while nodal displacements and an interpolation function are used in the Y direction. Governing equations can then be described as an eigensystem. Solving the eigensystem gives the wave numbers and wave structures corresponding to the resonant modes. Amplitude for each mode can be obtained from boundary conditions such as incident waveforms and transducer type. Waveforms at an arbitrary point are given as a superposition of these modes just like in normal mode expansion theory. Therefore, calculation by the semianalytical finite element method can assume an infinite plate without reflection wall and can deal with modal analysis. Thus the following calculations, in which a pure lamb mode propagates with no reflection, can easily be done.

Figures 1 through 4 show sample lamb waves of the fundamental symmetric and antisymmetric ($S0$ and $A0$) modes that are typically the most useful modes for NDT. Grid shift and shading represents displacement at the grid points. Excitation frequency / velocity regions are shown by gray circles in the phase and group dispersion curves. Figures 1 and 2 show the symmetric compressional wave structure of the $S0$ mode where the waveform is symmetric with respect to the center of the plate cross section. Figure 1 shows the nondispersive characteristic of the $S0$ mode in the frequency range indicated in the phase and group velocity dispersion curves. The total pulse duration varies very little as the pulse travels across the plate since the components of the waveform travel with about the same phase and group velocities. Figure 2 shows the increase in total pulse duration as the pulse travels across the plate. This occurs because of the dispersive character of wave propagation at that particular frequency. Since the pulse contains various frequency components and phase velocity is a function of frequency, the severe slope in the dispersion curves at the frequency selected leads to dispersion, hence pulse spreading. The propagation velocity of the center of the expanding pulse is equal to the traditional group

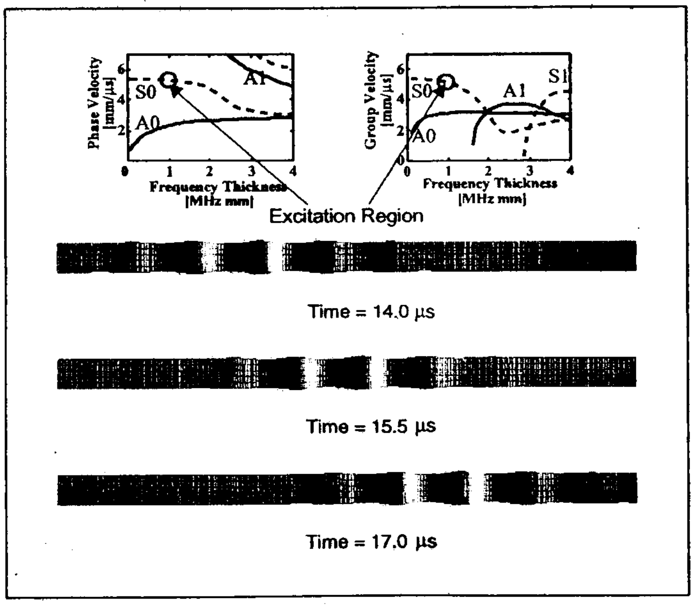

Figure 1 — S0 mode propagation (minimal dispersive region) in an aluminum plate (c_L = 6.3 mm/ms [0.25 in./ms], c_T = 3.1 mm/µs [0.1 in./ µs]), showing symmetric wave structure with respect to the center of the plate cross section and nondispersive wave propagation where the pulse duration does not vary as the pulse travels to the right. Grid shift and shading represent the real part of complex amplitude at the grid point.

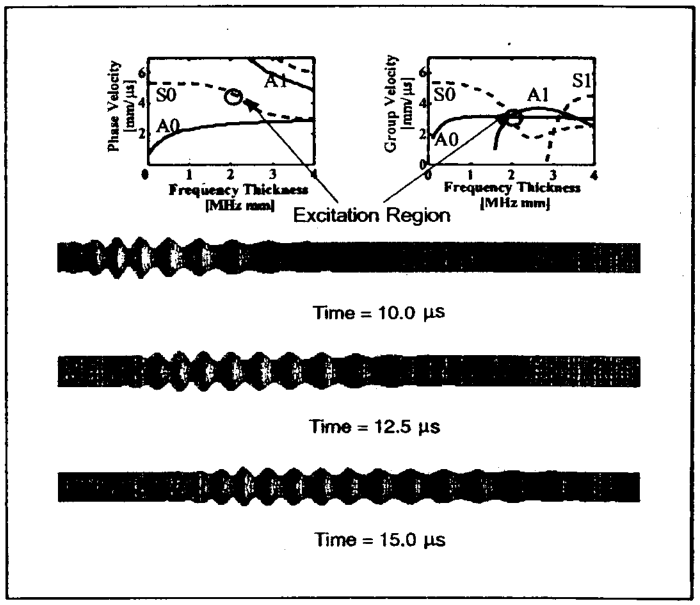

Figure 2 — S0 mode propagation (dispersive region) in an aluminum plate, showing the characteristic of dispersion where the pulse duration becomes longer as the pulse travels to the right.

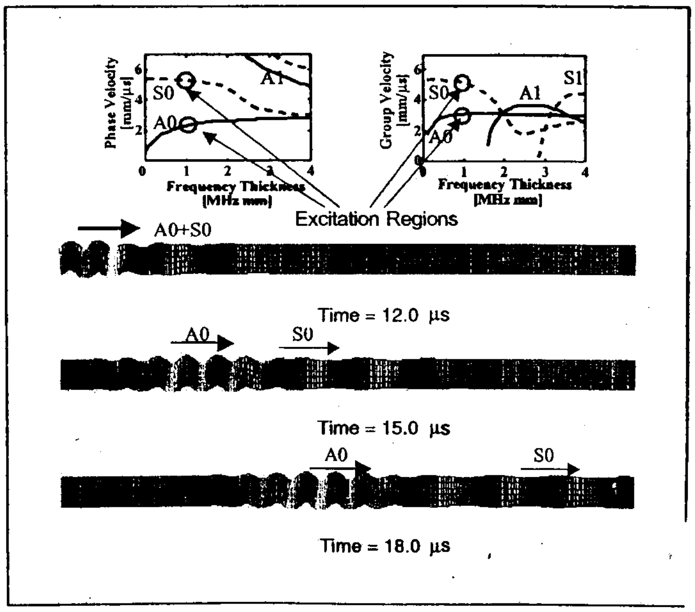

Figure 3 — A0 mode propagation (nondispersive region) in an aluminum plate, showing antisymmetric wave structure with respect to the center of the plate cross section and nondispersive wave propagation.

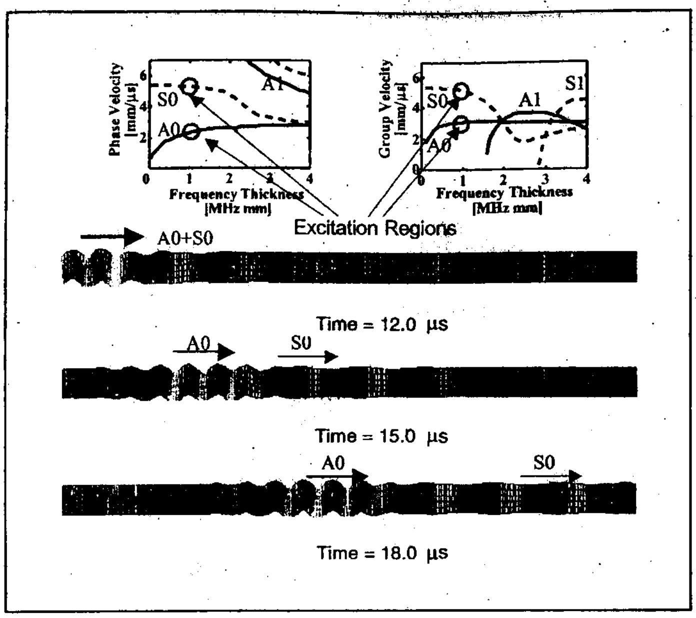

Figure 4 — Multimode propagation of S0 and A0 modes in an aluminum plate, showing that two possible modes are excited simultaneously, but separate as they travel across the plate.

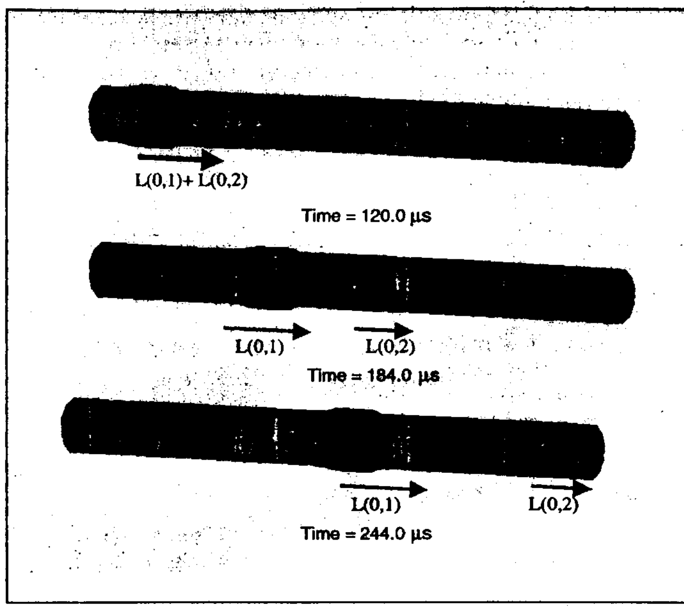

Figure 5 — Axisymmetric guided wave in a straight pipe, showing two possible axisymmetric waves with different velocities. Grid shift and shading represent the absolute value and the real part of the complex amplitude at the grid point, respectively.

velocity that is derived from the actual phase velocity value and slope in the dispersion curves. Figure 3 shows the antisymmetric wave structure and nondispersive character of wave propagation of the *A*0 mode at this particular frequency. Figure 4 shows the multimode existence of the *A*0 and *S*0 modes where the two modes separate as they travel in the material since they have different phase and group velocities. All of these four examples are predictable phenomena from phase and group dispersion curves. However, lamb wave simulation and visualization are very helpful to our understanding.

For more studies on lamb wave calculation, see Hayashi and Endoh (2000) and Hayashi et al. (2000) for animations of lamb wave generation mechanics and Datta et al. (1988), Guo and Cawley (1993), Liu and Achenbach (1995) and Hayashi and Kawashima (2002) for wave propagation in a layered plate.

GUIDED WAVES IN A PIPE

Calculations of axisymmetric mode propagation with no validation in the circumferential θ direction are considered a two dimensional problem in the thickness *r* and longitudinal Z directions. However, three dimensional calculations are inevitably required for nonaxisymmetric mode propagation. In a semianalytical finite element method for guided waves in a pipe, the use of orthogonal function exp ($in\theta$) in the circumferential direction and exp ($i\xi z$) in the longitudinal direction leads to an eigensystem; then, displacements and stresses can be obtained at each time (frequency) step as in the lamb wave calculation.

Figure 5 shows wave propagation when axisymmetric normal loading is applied on the left end of a straight pipe with an outer diameter of 88 mm (3.5 in.) and thickness 5 mm (0.2 in.). The shift of the grid point represents the absolute value of complex amplitude. Gray circles in the dispersion curves of Figure 6 represent the frequency range of the applied dynamic loading. In this frequency range, axisymmetric modes consist of longitudinal modes *L*(0, 1) and *L*(0, 2) and a torsional mode *T*(0, 1), but since these waves are excited by the normal loading on the surface of the pipe, the torsional mode, a type of shear wave, is not excited. Two propagating modes with different velocities are shown in the visualization results as seen in the group velocity dispersion curves; the faster one being *L*(0, 2) mode and the slower one *L*(0, 1). Similar to lamb wave propagation in Figure 4, the two modes are separate as they propagate to the right.

Reflections from discontinuities could be simulated and visualized by performing the semianalytical finite element method.

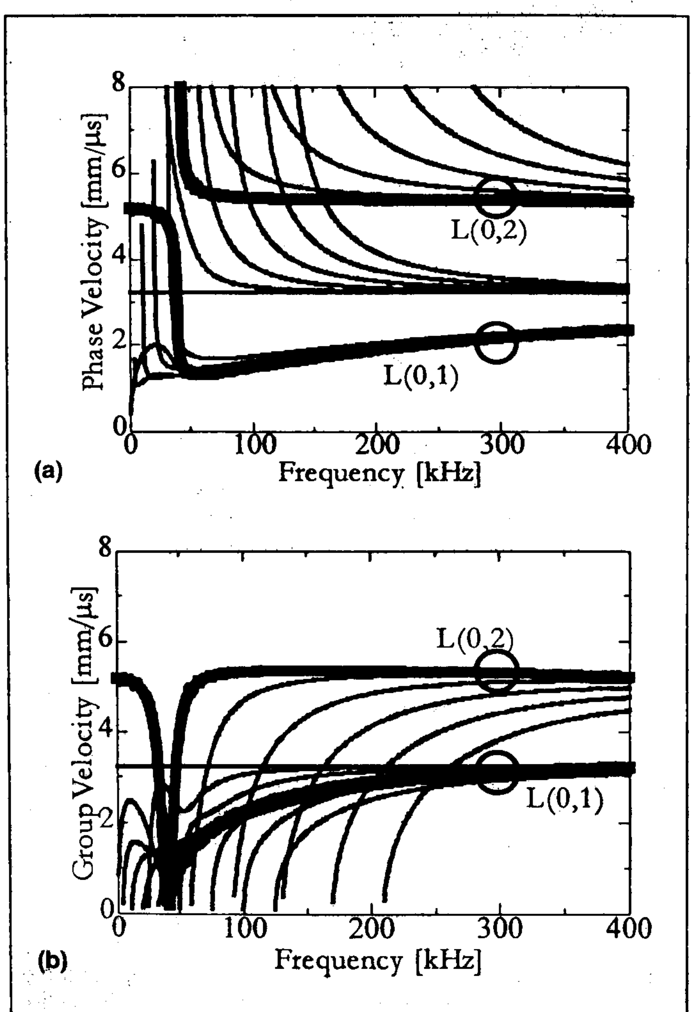

Figure 6 — Dispersion curve for a steel pipe. Outer diameter and thickness are 88 mm (3.5 in.) and 5 mm (0.2 in.), respectively. Bold lines represent all possible longitudinal modes in this frequency range. Circles show excitation regions for the simulation in Figure 5.

Partial loading responsible for nonaxisymmetric flexural wave propagation in the pipe could also be studied. Figure 7 presents one example of flexural mode studies demonstrating focusing effects in a straight pipe (Hayashi et al., in press (a); Rose, 2002). Using time delays and amplitudes predetermined by a focusing algorithm (Li and Rose, 2001), flexural mode focusing can be seen at the designated focal point in a straight pipe at 400 mm (15.7 in.). Some other modes are also shown in the pipe.

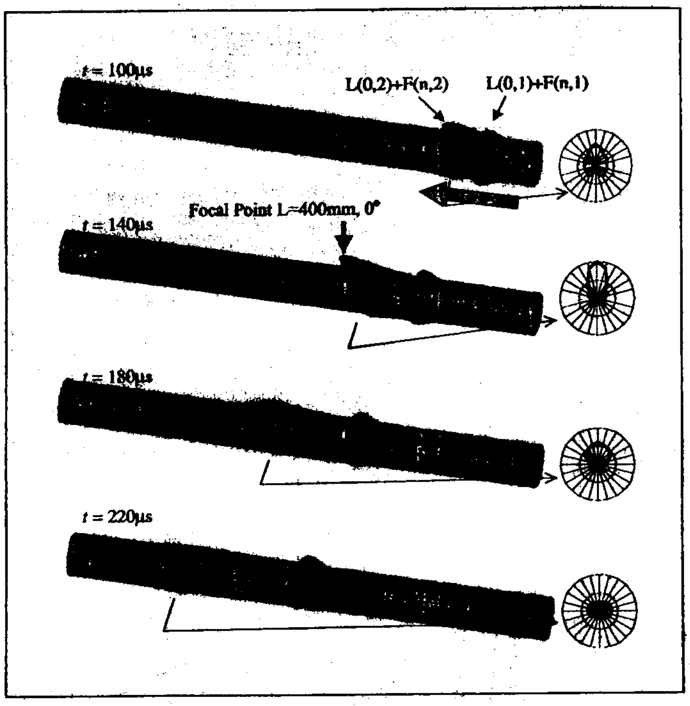

Figure 7 — Guided wave focusing with time delays and amplitudes controlled independently by eight angle beam transducers. Time delays and amplitudes are determined for focusing at 400 mm (15.7 in.) away from transmitters by the focusing algorithm. Circumferential profiles are shown in the circle charts.

Elbow testing and pipe testing beyond the elbow have become major issues due to their complex wave mechanics, such as mode conversion at the elbow and subsequent nonaxisymmetric wave propagation. Recently, many experimental and practical work efforts have been completed (Alleyne and Cawley, 1997; Kwun and Dynes, 1998; Li and Rose, 2001) in which discontinuity detection potential using guided waves is presented. More theoretical and numerical studies are necessary to use guided waves efficiently and to improve guided wave testing. The semianalytical finite element method could also be used to study guided wave calculations in a pipe with an elbow. Figure 8 shows axisymmetric input and subsequent nonaxisymmetric wave propagation beyond the elbow region. As expected, the axisymmetric wave breaks up at the elbow and very complicated guided waves propagate beyond the elbow. This creates an inefficiency in pipe testing using axisymmetric guided waves. The semianalytical finite element method could be used to study ways of generating axisymmetric waveforms and focusing effect beyond the elbow region, perhaps by utilizing a phased array input of partial loading segments around the circumference.

CONCLUDING REMARKS

Simulation and visualization of guided wave propagation in a plate and a pipe were carried out using a semianalytical finite element method. Visualization results of lamb wave propagation of the fundamental modes explain lamb wave structure, dispersion and multimode existence very well. These kinds of visualization results are very helpful to our understanding of lamb wave mechanics, which leads to more advanced applications. The semianalytical finite element method tool can now be used on a variety of new, useful and practical problems involving guided waves in a straight

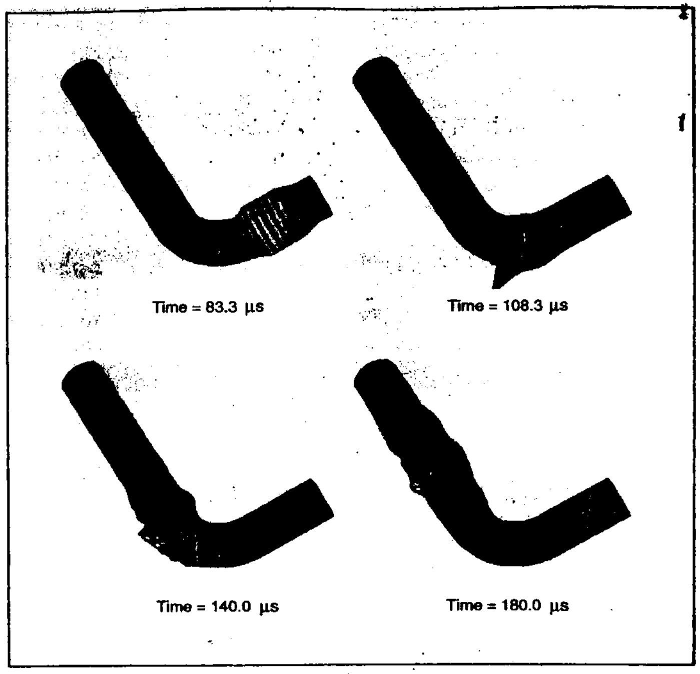

Figure 8 — Axisymmetric wave input in a pipe with elbow, showing the wave breakup and mode conversion at an elbow and subsequent nonaxisymmetric wave propagation.

pipe and in a pipe with an elbow, as a result of mode conversion to flexural modes.

ACKNOWLEDGMENTS

We thank Plant Integrity, Ltd., of Cambridge, England, for their partial support of this work, with special thanks to Peter Mudge for technical discussion. Thanks are also given to Koichiro Kawashima.

All calculation results shown in this paper were obtained through the use of Visual Fortran (Compaq) on Microsoft Developer studio for calculations and Micro AVS (KGT, Inc.), simplified software of Advanced Visualization Systems for visualization.

REFERENCES

Achenbach, J.D., *Wave Propagation in Elastic Solids,* New York, North-Holland Publishing Company, 1984.

Al-Nassar, Y.N., S.K. Datta and A.H. Shah, "Scattering of Lamb Waves by a Normal Rectangular Strip Weldment," *Ultrasonics,* Vol. 29, 1991, pp. 125-132.

Alleyne, D.N. and P. Cawley, "Long Range Propagation of Lamb Wave in Chemical Plant Pipework," *Materials Evaluation,* Vol. 45, No. 4, 1997, pp 504-508.

Auld, B.A., *Acoustic Fields and Waves in Solids,* Vols. 1-2, second edition, Florida, Kreiger, 1990.

Cho, Y. and J.L. Rose, "A Boundary Element Solution for a Mode Conversion Study on the Edge Reflection of Lamb Waves," *Journal of the Acoustical Society of America,* Vol. 99, 1996, pp. 2097-2109.

Datta, S.K., A.H. Shah, R.L. Bratton and T. Chakraborty, "Wave Propagation in Laminated Composite Plates," *Journal of the Acoustical Society of America,* Vol. 88, No. 6, 1988, pp. 2020-2025.

Demma, A., P. Cawley and M.J.S. Lowe, "Mode Conversion of Longitudinal and Torsional Guided Modes Due to Pipe Bends," *Review of Progress in Quantitative Nondestructive Evaluation,* Vol. 20A, D.O. Thompson and D. Chimenti, eds., New York, American Institute of Physics, 2001, pp. 172-17

Ditri, J. and J.L. Rose, "Excitation of Guided Elastic Wave Modes in Hollow Cylinders by Applied Surface Tractions," *Journal of Applied Physics,* Vol. 72, No. 7, 1992, pp. 2589-2597.

Galán, J.M. and Abascal R., "Numerical Simulation of Lamb Wave Scattering in Semi-infinite Plates," *International Journal for Numerical Methods Engineering,* Vol. 53, 2002, pp. 1145-1173.

Graff, K.F., *Wave Motion in Elastic Solids,* New York, Dover, 1991.

Guo, N. and P. Cawley, "The Interaction of Lamb Waves with Delaminations in Composite Laminates," *Journal of the Acoustical Society of America,* Vol. 94, 1993, pp. 2240-2246.

Harumi, K., "Computer Simulation of Ultrasonics in a Solid," *Nondestructive Testing International,* Vol. 19, 1986, pp. 315-332.

Hayashi, T. and S. Endoh, "Calculation and Visualization of Lamb Wave Motion," *Ultrasonics*, Vol. 38, 2000, pp. 770-773.
Hayashi, T., K. Kawashima and S. Endoh, "The Generation and Detection of Fundamental Lamb Modes in Plastic Plates by Air-coupled Transducers," *Review of Progress in Quantitative Nondestructive Evaluation*, Vol. 20A, D.O. Thompson and D.E. Chimenti, eds., New York, American Institute of Physics, 2000, pp. 105-110.
Hayashi, T. and Kawashima K., "Multiple Reflections of Lamb Waves at a Delamination," *Ultrasonics*, Vol. 40, 2002, pp. 193-197.
Hayashi, T., Z. Sun, J.L. Rose and K. Kawashima, "Analysis of Flexural Mode Focusing by a Semi-analytical Finite Element Method," *Journal of the Acoustical Society of America*, in press (a).
Hayashi, T., W. Song and J.L. Rose, "Guided Wave Dispersion Curves for a Bar with an Arbitrary Cross-section, a Rod and Rail Example," *Ultrasonics*, in press (b).
Kino, C.S., *Acoustic Waves: Devices, Imaging and Digital Signal Processing*, New Jersey, Prentice Hall, 1987.
Kwun, H. and H. Dynes, "Long Range Guided Wave Inspection of Pipe Using the Magnetostrictive Sensor Technology — Feasibility of Defect Characterization," *International Society for Optical Engineering (SPIE) on Nondestructive Evaluation of Utilities and Pipelines II*, Vol. 65, 1998, pp. 424-430.
Li, H.U. and K. Negishi, "Visualization of Lamb Mode Patterns in a Glass Plate," *Ultrasonics*, Vol. 32, No. 4, 1994, p. 243.
Li, J. and J.L. Rose, "Excitation and Propagation of Non-axisymmetric Guided Waves in a Hollow Cylinder," *Journal of the Acoustical Society of America*, Vol. 109, 2001, pp. 457-464.
Liu, G.R. and J.D. Achenbach, "Strip Element Method to Analyze Wave Scattering by Cracks in Anisotropic Laminated Plates," *Journal of Applied Mechanics*, Vol. 62, 1995, pp. 607-613.
Rose, J.L., "Guided Wave Ultrasonic Pipe Inspection — The Next Generation," 8th European Conference on Non-destructive Testing, Barcelona, Spain, June 2002.
Rose, J.L., *Ultrasonic Waves in Solid Media*, Cambridge, Cambridge University Press, 1999.
Sanderson, R. and S. Smith, "The Application of Finite Element Modelling to Guided Ultrasonic Waves in Rails," *Insight*, Vol. 44, No. 6, 2002, pp. 359-363.
Taweel, H., S.B. Dong and M. Kazic, "Wave Reflection from the Free End of a Cylinder with an Arbitrary Cross-section," *International Journal of Solids and Structures*, Vol. 37, 2000, pp. 1701-1726.
Viktorov, I.A., *Rayleigh and Lamb Waves — Physical Theory and Applications*, New York, Plenum Press, 1967.
Yamawaki, H. and T. Saito, "Numerical Calculation of Surface Waves Using New Nodal Equations," *Nondestructive Testing and Evaluation*, Vol. 8/9, 1992, pp. 379-389.
Zhang, S.Y., J.Z. Shen and C.F. Ying, "The Reflection of the Lamb Wave by a Free Plate Edge: Visualization and Theory," *Materials Evaluation*, Vol. 46, 1988, pp. 638-641.
Zhuang, W., A.H. Shah and S.B. Dong, "Elastodynamic Green's Function for Laminated Anisotropic Circular Cylinders," *Journal of Applied Mechanics*, Vol. 66, 1999, pp. 665-674.